Hilbert and Einstein's General Theory of Relativity: Two Communications on the Foundations of Physics

Katherine Brading, University of Notre Dame

The end of 1915 saw David Hilbert and Albert Einstein involved in a frenetic period of activity out of which emerged Einstein's General Theory of Relativity. The focus of my talk is a re-assessment of Hilbert's project in physics at this time: its motivations, goals, and achievements.

In November and December 1915, Hilbert gave two presentations to the Royal Göttingen Academy of Sciences under the common title 'The Foundations of Physics'. Distinguished as 'First Communication' (Hilbert, 1915b) and 'Second Communication' (Hilbert, 1917), two papers (or 'notes', as they are widely known) eventually appeared in the *Nachrichten* of the Academy. I will discuss these papers, in conjunction with the recently discovered printer's proofs of the first communication (Hilbert, 1915a; see Corry, Renn and Stachel, 1997).

Until recently, these two communications have been judged largely on the assumption that Hilbert and Einstein shared the same goal of finding generally covariant field equations for gravitation. From this perspective, they present a mixed record of achievement, ranging from genuine insight (the Riemann scalar as the suitable invariant for the gravitational action) through incomprehension (Hilbert's interpretation of electromagnetism as a consequence of gravitation) to abject failure (attachment to the untenable electromagnetic theory of matter of Gustav Mie). The usual implication is that Hilbert's principal intent in November 1915 was to arrive at a theory of gravitation based on the principle of general covariance in one blinding flash, masterfully wielding an arsenal of axiomatized advanced mathematics. Thus arose 'the legend of a royal road to general relativity' (Renn and Stachel, 1999, p.1) through the axiomatic method.

The First Communication, which quickly entered the canon of classical General Relativity, became the object of renewed scholarly following the discovery of the proofs noted above. With the exception of Renn and Stachel (1999), the Second Communication has not been given the same detailed reconsideration. However, the analysis of the Second Communication by Renn and Stachel seriously misrepresents its aims, content, and significance, and also its links to the First Communication. This is due to the perspective taken by the authors: they assess Hilbert's work against the goals of Einstein's project. I will urge that more careful attention to the historical and philosophical context of the two communications leads to strikingly different conclusions. In particular, I will show that once viewed in its proper context, Hilbert's Second Communication is a natural continuation of his First Communication, that it contains important and interesting further developments of that project, and that it sheds needed illumination on Hilbert's assessment of the epistemological novelty posed by a generally covariant physics.

I will discuss Hilbert's two communications against the backdrop of his own approach to the foundations of physics, in which finding the specific form of the generally covariant field equations is not the main target. This backdrop includes Hilbert's long-standing interest in physics, as documented by Corry (2004), along with his axiomatic method. The intimate relationship between these two is emphasized by Corry; however, I will develop a more detailed account of the aims and methods of Hilbert's axiomatic method, and show that this is crucial for understanding Hilbert's 1915-1917 work on generally covariant physics.

Hilbert arrived at important results prior to Einstein's formulation of the canonical version of the field equations, some of which are independent of the precise form of these equations. These include consequences for the relationship between electromagnetism and gravitation, and the realization that any generally covariant theory raises questions about causality, in the precise sense of Cauchy determination. This is *not* the same problem as that found in Einstein's 'hole argument' – something that, I argue, never confused Hilbert. That said, Hilbert adapted Einstein's original solution to the 'hole

argument' – the use of energy conservation to restrict the covariance properties of his field equations – to arrive at a solution to his own causality problem. We know this from the proofs. The advent of the field equations required Hilbert to revisit some of his earlier results, and the original solution does not appear in the published version of the First Communication. What we find in the Second Communication is, I argue, Hilbert's revised solution. Thus, in my view, the proofs of the First communication provide a vital link between the published versions of the First and Second communications. One aim of my talk is to display the continuity of Hilbert's project between his First and Second communications, in terms of both the problems tackled and the methodology brought to bear in the search for solutions.

The pursuit of Hilbert's original program in the light of the Einstein field equations produced further significant conclusions. The tension between general covariance and causality is identified in the First Communication, via the axiomatic method, as Theorem I (the 'Leitmotiv' of his theory). This tension is given a deeper and richer treatment in the Second Communication, once again explicitly employing the axiomatic method. Hilbert understood the challenge as epistemological, and sought to offer a solution within a broadly Kantian framework. His diagnosis is that the principle of causality should be dropped as a principle of physical objectivity, with general covariance being the sole necessary and sufficient condition for physical objectivity. As a result, Hilbert argued, we are driven to *reformulate* the causality principle, and he offers just such a reformulation.

This talk is both historical and philosophical. It seeks to re-tell the story of Hilbert's involvement in generally covariant physics, 1915-1917, from Hilbert's perspective. I will show that (a) Hilbert's two communications should be regarded as part of a wider research program within the overarching framework of 'the axiomatic method' (as Hilbert expressly stated was the case), and (b) the second communication is a fine and coherent piece of work within this framework. In re-telling the story, I also seek to clarify the philosophical context, questions, and answers offered by Hilbert during this period. This talk is based on joint research with Tom Ryckman.